辐射的基本概念

各向同性点光源 $F \propto \frac{1}{r^2}$

specific intensity $I_\nu(t,\vec{x},\hat{n},\nu) = \frac{dE}{dt\,dA\,d\Omega\,d\nu}$

- constant along rays in free space (individual)
- not contradictory with 平方反比, $\frac{dF}{d\Omega} = \pi B\left(\frac{R}{r}\right)^2$
- 在表面 $\frac{dF}{d\Omega} = \pi B$

辐射转移 发射 $j_\nu = \frac{dE}{dV\,dt\,d\Omega\,d\nu}$

吸收(受激发射) $\alpha_\nu = -\frac{1}{I_\nu}\frac{dI_\nu}{ds} = n\sigma_\nu = \kappa\rho$ $(\sqrt{\sigma_\nu} \ll d)$

方程 $\frac{dI_\nu(s)}{ds} = j_\nu(s) - \alpha_\nu(s) I_\nu(s)$, $\tau = \int_0^s \alpha_\nu(s')ds'$

$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$, $S_\nu = \frac{j_\nu(s)}{\alpha_\nu(s)}$

形式解 $I_\nu(s) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu-\tau_\nu')} S_\nu(\tau_\nu')d\tau_\nu'$

mean free path $l_\nu = \frac{1}{\alpha_\nu}$

黑体辐射, 热辐射

辐射场是平衡于物体发出的辐射

黑体辐射 $I_\nu = B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$ ↑ neutrino for '+' hw (1.7)

热辐射 $S_\nu = I_\nu^{bb} = B_\nu$

$$B_\nu(T)\begin{cases}\text{低频 } h\nu \ll kT & B_\nu(T) \simeq \frac{2\nu^2}{c^2}kT \\ \text{高频 } h\nu \gg kT & B_\nu(T) \simeq \frac{2h\nu^3}{c^2}e^{-h\nu/kT}\end{cases}$$

$\frac{\partial B_\nu(T)}{\partial T} > 0$, $\frac{h\nu_{max}}{kT} = 2.82$ $\left(\frac{\nu_{max}}{T} = 5.88\times10^{10}\,\mathrm{Hz\cdot K^{-1}}\right)$

$\int F_\nu d\nu = \sigma T^4 = \frac{2\pi^5 k^4}{15c^2h^3}T^4$

亮温度 $T_b = \frac{c^2}{2\nu^2 k}I_\nu$ ⇒ $\frac{dT_b}{d\tau_\nu} = -T_b + T$

色温度 (ν_{max})

$T = T(0)e^{-\tau_\nu} + T_s(1-e^{-\tau_\nu})$

有效温度 ($F = \sigma T^4$)

Einstein A, B 系数

吸收 $B_{12}\bar{J}$

受激/自发辐射 $B_{21}\bar{J}$ / A_{21} (↳ <0 for neutrino)

$n_1 B_{12}\bar{J} = n_2 A_{21} + n_2 B_{21}\bar{J}$
+
$\bar{J} = B_\nu$
$\frac{n_1}{n_2} = \frac{g_1}{g_2}e^{h\nu/kT}$

Einstein 关系: $g_1 B_{12} = g_2 B_{21}$, $A_{21} = \frac{2h\nu^3}{c^2}B_{21}$

$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{A_{21}/B_{21}}{\frac{n_1 B_{12}}{n_2 B_{21}}-1} = \frac{2h\nu^3/c^2}{\frac{g_2 n_1}{g_1 n_2}-1}$ (小于1时,粒子数反转)

$$\begin{cases} j_\nu = \frac{h\nu_0}{4\pi}n_2 A_{21}\phi(\nu) \\ \alpha_\nu = \frac{h\nu_0}{4\pi}\phi(\nu)(n_1B_{12} - n_2B_{21})\end{cases}$$

经典的辐射理论

maxwell eq. $\nabla\cdot\vec{D} = 4\pi\rho$ $\nabla\cdot\vec{B} = 0$

$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$ $\nabla\times\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}$

$$\begin{cases}\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H} \\ u_{em} = \frac{1}{8\pi}\left(\epsilon E^2 + \frac{B^2}{\mu}\right) \\ \vec{P}_{em} = \vec{S}/c^2\end{cases}$$

平面波解 ($\vec{E}$, $\vec{B}$, $\hat{k}(\hat{n})$)

$$\begin{cases}\omega = ck \\ E_0 = B_0 \\ v_{ph} = v_{gr} = c \\ |\langle\vec{S}\rangle| = \frac{c}{8\pi}|E_0|^2 \quad \langle u\rangle = \frac{|\langle\vec{S}\rangle|}{c}\end{cases}$$

$\langle \mathrm{Re}A(t)\,\mathrm{Re}B(t)\rangle = \frac{1}{2}\mathrm{Re}(AB^*)$ 时均

非平面波 $\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}E(t)e^{i\omega t}dt$

$E(t) = \int_{-\infty}^{\infty}\hat{E}(\omega)e^{-i\omega t}dt$

$\frac{dW}{dA} = \int \underbrace{\frac{c}{4\pi}E^2(t)}_{\frac{dW}{dt\,dA}}dt = \int \underbrace{c|\hat{E}(\omega)|^2}_{\frac{dW}{d\omega\,dA}}d\omega$ 对于正频

$T\cdot\Delta\omega \doteq 1$ $= \frac{dW}{dt\,d\Omega\,R_0^2}$

$\frac{d\sigma}{d\Omega} = \frac{dW}{dt\,d\Omega}\frac{1}{\langle S\rangle}$

偏振 (polarization)

$E = \hat{x}\,\mathcal{E}_1 e^{i(\phi_1-\omega t)} + \hat{y}\,\mathcal{E}_2 e^{i(\phi_2-\omega t)}$

$\mathrm{Re}E = \hat{x}'\cos\omega t\cdot\mathcal{E}_0\cos\beta - \hat{y}'\sin\omega t\cdot\mathcal{E}_0\sin\beta$

Stokes parameters

$$\left.\begin{aligned} I &= \mathcal{E}_1^2 + \mathcal{E}_2^2 = \mathcal{E}_0^2 \\ Q &= \mathcal{E}_1^2 - \mathcal{E}_2^2 = \mathcal{E}_0^2\cos2\beta\cos2\chi \\ U &= 2\mathcal{E}_1\mathcal{E}_2\cos(\phi_1-\phi_2) = \mathcal{E}_0^2\cos2\beta\sin2\chi \\ V &= 2\mathcal{E}_1\mathcal{E}_2\sin(\phi_1-\phi_2) = \mathcal{E}_0^2\sin2\beta\end{aligned}\right\} I^2 = Q^2 + U^2 + V^2$$

χ 是 $\hat{x}$ 到 $\hat{x}'$ 的转角

Some more def

$I_\nu \to \int I_\nu\cos\theta\,d\Omega = F_\nu$ (flux)

$I_\nu \to \frac{1}{4\pi}\int I_\nu\,d\Omega = J_\nu$ (mean Intensity)

$I_\nu \to \frac{I_\nu}{c} = u_\nu(\hat{n})$ (energy density)

电磁势

$\vec{B} = \nabla\times\vec{A}$ $\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$ Lorentz 条件 $\nabla\cdot\vec{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0$

用电磁势表示的 Maxwell 方程

$$\begin{cases}\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -4\pi\rho \\ \nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{j}\end{cases}$$

解 $\phi(\vec{r},t) = \int\frac{d^3r'}{|\vec{r}-\vec{r}'|}[\rho]$

$\vec{A}(\vec{r},t) = \int\frac{d^3r'}{|\vec{r}-\vec{r}'|}[\vec{j}]$ (推迟势)

运动粒子的辐射

$\phi(\vec{r},t) = \left[\frac{q}{\kappa R}\right]$

↳ $\kappa(t') = 1 - \frac{\hat{n}(t')}{c}\cdot\vec{u}(t')$ $\beta\to1$, $\kappa\to0$ along $\vec{u}$ (beam effect)

$\vec{A}(\vec{r},t) = \left[\frac{q\vec{u}}{\kappa R}\right]$

$\vec{E}(\vec{r},t) = q\left[\frac{(\hat{n}-\vec{\beta})(1-\beta^2)}{\kappa^3R^2}\right] + \frac{q}{c}\left[\frac{\hat{n}}{\kappa^3R}\times\{(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\}\right]$

$\vec{B}(\vec{r},t) = \hat{n}\times\vec{E}(\vec{r},t)$

Case 1: particle moving at constant v

Case 2: change of velocity (stopped)

$\frac{dW}{d\omega\,d\Omega} = c|R\hat{E}(\omega)|^2 = \frac{q^2}{4\pi^2c}\left|\int[\hat{n}\times\{(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\}\kappa^{-3}]_{t'}e^{i\omega t}dt\right|^2$

$= \frac{q^2\omega^2}{4\pi^2c}\left|\int\hat{n}\times(\hat{n}\times\vec{\beta})\exp\left(i\omega\left(t' - \frac{\hat{n}\cdot\vec{r}_0(t')}{c}\right)\right)dt'\right|^2$

Case 3: Non-relativistic particle

$E_{rad}/E_{vel} = \beta\frac{R}{\lambda}$ $\begin{cases}R \ll \lambda & \text{(Near Zone)} \\ R \gg \lambda\beta^{-1} & \text{(Far Zone)}\end{cases}$

Larmor's formula

$\vec{E}_{rad} = \frac{q}{Rc^2}\hat{n}\times(\hat{n}\times\dot{\vec{u}})$ (hw3.1) 磁偶极 $B_0 = \frac{2m}{R^3}$

$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B}$

$\frac{dW}{dt\,d\Omega} = \frac{q^2\dot{u}^2\sin^2\theta}{4\pi c^3}$ $\frac{dW}{dt} = \frac{2q^2\dot{u}^2}{3c^3}$

偶极近似 $\tau \sim \frac{1}{\nu} \sim \frac{\lambda}{c} \gg \frac{L}{c}$

即 $u \ll c$

$\vec{E}_{rad} = \frac{\hat{n}\times(\hat{n}\times\ddot{\vec{d}})}{c^2R_0}$ $\vec{d} = \sum_i q_i\vec{r}_i$

$\frac{cR_0^2}{4\pi}|E(t)|^2 = \frac{dP}{d\Omega} = \frac{|\ddot{\vec{d}}|^2\sin^2\theta}{4\pi c^3}$ $dW/dt = 2|\ddot{\vec{d}}|^2/3c^3$

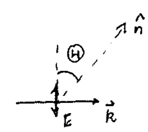
$\hat{E}(\omega) = -\frac{\omega^2}{c^2R_0}\hat{d}(\omega)\sin\theta$

$\frac{dW}{d\omega\,d\Omega} = \frac{\omega^4}{c^3}|\hat{d}(\omega)|^2\sin^2\theta$ $\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2$

Thomson Scattering

入射电场 $\vec{E} = \hat{\epsilon}E_0e^{i(\omega t + \vec{k}\cdot\vec{r})}$

$\vec{d} = e\vec{r}$, $m\ddot{\vec{r}} = e\vec{E}$



$\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{e^4E_0^2\sin^2\Theta}{8\pi m^2c^3}$ $\langle P\rangle = \frac{e^4E_0^2}{3m^2c^3}$

$\frac{d\sigma}{d\Omega} = \frac{\langle dP/d\Omega\rangle}{\langle S\rangle} = r_0^2\sin^2\Theta$, $r_0 = \frac{e^2}{mc^2}$, $\sigma = \frac{8\pi}{3}r_0^2$

($\langle S\rangle$ ↳ $\frac{c}{8\pi}E_0^2$) size of point charge

入射未偏振的辐射时 散射光是 polarized 的.

$\left(\frac{d\sigma}{d\Omega}\right)_{unpol} = \frac{1}{2}r_0^2(1+\sin^2\Theta)$ ↑ 散射角

$\sigma_{unpol} = \frac{8}{3}\pi r_0^2 = \sigma_{pol}$

degree of polarization $\Pi = \frac{1-\cos^2\alpha}{1+\cos^2\alpha}$ $\begin{cases}\alpha = 0. & 0\% \\ \alpha = \frac{\pi}{2}. & 100\%\end{cases}$

Scattering $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + \alpha_\nu B_\nu - \sigma_\nu(I_\nu - J_\nu) = -(\alpha_\nu+\sigma_\nu)(I_\nu - S_\nu)$

$J_\nu = \int I_\nu\frac{d\Omega}{4\pi}$ $S_\nu = \frac{\alpha_\nu B_\nu + \sigma_\nu J_\nu}{\alpha_\nu + \sigma_\nu}$

$\epsilon_\nu = \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu}$

$l_* = l\sqrt{N} = l\frac{1}{\sqrt{\epsilon}} = [\alpha_\nu(\alpha_\nu+\sigma_\nu)]^{-1/2}$

$\tau_* = \frac{L}{l_*}$ ⇒ $\begin{cases}\tau_* \ll 1. & L_\nu = 4\pi\alpha_\nu B_\nu V \\ \tau_* \gg 1 & L_\nu = 4\pi\alpha_\nu B_\nu A l_* = 4\pi\sqrt{\epsilon_\nu}B_\nu A\end{cases}$

$B = \frac{2m}{R^3}$ for dipole

相对论运动学 $x^\mu, x_\mu, \eta_{\mu\nu}, s^2 = x_\mu x^\mu, T^{\mu\nu}$, $\det \Lambda = 1$, $\{\Lambda^0{}_0 \geq 1$

坐标变换 $x'^\mu = \Lambda^\mu{}_\nu x^\nu$, $x'_\mu = x_\nu \tilde{\Lambda}^\nu{}_\mu$, $\tilde{\Lambda}^\mu{}_\sigma \Lambda^\sigma{}_\nu = \delta^\mu_\nu$

四速 $u^\mu = \frac{dx^\mu}{d\tau} = \gamma_u (c, \vec{u})$, $\gamma_u = (1-\beta^2)^{-1/2}$

波矢 $k^\mu = (\frac{\omega}{c}, \vec{k})$　电流 $j^\mu = (\rho c, \vec{j})$　$\partial_\mu j^\mu = 0$

规范势 $A^\mu = (\phi, \vec{A})$　$\partial_\mu \partial^\mu A^\nu = -\frac{4\pi}{c} j^\nu$ (Maxwell eq.)

$\partial_\mu A^\mu = 0$ (Lorentz gauge)

速度变换 $\frac{dt'}{dt} = \frac{1}{\gamma\sigma}$, $\sigma = 1 + \frac{v u_x'}{c^2}$

$$\begin{cases} u_x = \frac{1}{\sigma}(v + u_x') \\ u_y = u_y'/\gamma\sigma \\ u_z = u_z'/\gamma\sigma \end{cases}$$

四加速 $a^\mu = \frac{du^\mu}{d\tau}$

场强张量 $F_{\mu\nu} = \begin{pmatrix} 0 & & & \\ E_x & 0 & B_z & -B_y \\ E_y & & 0 & B_x \\ E_z & & & 0 \end{pmatrix}$ 反对称

Lorentz变换下 $\vec{E}, \vec{B}$ 的变换 $E'_\parallel = E_\parallel$, $B'_\parallel = B_\parallel$

$\vec{E}'_\perp = \gamma(\vec{E}_\perp + \vec{\beta} \times \vec{B})$　$\vec{B}'_\perp = \gamma(\vec{B}_\perp - \vec{\beta} \times \vec{E})$

$\partial^\nu F_{\mu\nu} = \frac{4\pi}{c} j_\mu$ (Maxwell)　$\partial_{[\sigma} F_{\mu\nu]} = 0$

不变量 $F^{\mu\nu} F_{\mu\nu} = 2(|\vec{B}|^2 - |\vec{E}|^2)$, $\det F_{\mu\nu} = (\vec{E} \cdot \vec{B})^2$

相对论力学

四动量 $p^\mu = m u^\mu = (\frac{E}{c}, \vec{p})$,　四力 $F^\mu = m a^\mu = m \frac{dp^\mu}{d\tau}$

洛伦兹力 $F^\mu = \frac{e}{c} F^\mu{}_\nu u^\nu$, $\vec{F} = e(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$, $F^\mu u_\mu = 0$

相对论带电粒子的辐射 (Lamour + Boost)

$\mu = \cos\theta$,　$\mu = \frac{\mu' + \beta}{1 + \beta\mu'}$,　$\sin^2\theta' = \frac{\sin^2\theta}{\gamma^2(1-\beta\mu)^2}$ (与 $\vec{v}$ 的角度),　$d\mu = \frac{d\mu'}{\gamma^2(1+\beta\mu')^2}$

$P = \frac{2q^2}{3c^3} \vec{a}' \cdot \vec{a}' = \frac{2q^2}{3c^3} \gamma^4 (a_\perp^2 + \gamma^2 a_\parallel^2)$

$a'_\parallel = \gamma^3 a_\parallel$, $a'_\perp = \gamma^2 a_\perp$　（ $\vec{a} = \frac{d^2\vec{x}}{dt^2}$ ）

Ex 4.14 Note: $F_\parallel = F'_\parallel$, $F_\perp = \gamma^{-1} F'_\perp$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} \frac{(\gamma^2 a_\parallel^2 + a_\perp^2)}{(1-\beta\mu)^4} \sin^2\Theta'$$

（ Θ' 与 $\vec{a}'$ 的角度）

韧致辐射 (free-free emission)

同种粒子, 不会有偶极辐射

single speed e^-: $\frac{dW}{d\omega}(b) = \begin{cases} 8z^2 e^6 / 3\pi c^3 m^2 v^2 b^2, & b \lesssim \frac{v}{\omega} \\ 0, & \text{otherwise} \end{cases}$

单速电子流: $\frac{dW}{d\omega\, dV dt} = \frac{16 e^6}{3 c^3 m^2 v} n_e n_i z^2 \ln \frac{b_{max}}{b_{min}} \rightarrow \frac{v}{\omega} = \frac{16\pi e^6}{3\sqrt{3} c^3 m^2 v} n_e n_i z^2 g_{ff}(v, \omega)$

量子化 / $\Delta p \Delta x = \frac{\hbar}{2}$　gaunt factor

Thermal bremsstrahlung: $\frac{dW}{d\omega dV dt}(T, \omega) = \frac{32\pi e^6}{3 m c^3} \left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} z^2 n_e n_i e^{-\frac{h\nu}{kT}} \bar{g}_{ff}$

$\frac{dW}{dt dV} = \left(\frac{2\pi kT}{3m}\right)^{1/2} \frac{32\pi e^6}{3 h m c^3} z^2 n_e n_i \bar{g}_B$

$= 1.4 \times 10^{-27} T^{1/2} n_e n_i z^2 \bar{g}_B$

$\bar{g}_{ff} = \frac{h\nu}{kT} \int_1^\infty g_{ff}(v, \omega) e^{-\frac{h\nu}{kT}(x-1)} x\, dx \sim O(1)$

$\bar{g}_B = \int \bar{g}_{ff} e^{-\frac{h\nu}{kT}} d(\frac{h\nu}{kT})$　$v_{min} = \sqrt{2h\nu / m}$, $x = \frac{v}{v_{min}}$

$\epsilon_\nu^{ff} = \frac{dW}{dV dt d\nu} = 6.8 \times 10^{-38} z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$ erg·s⁻¹·cm⁻³ Hz⁻¹　($\bar{g}_{ff} \sim 1.1\text{–}1.5$)

Free-Free absorption (逆过程)

$j_\nu^{ff} = \frac{dW}{dt dV d\nu} \cdot \frac{1}{4\pi}$,　$\alpha_\nu^{ff} = \frac{j_\nu^{ff}}{B_\nu(T)} = 3.7 \times 10^8 T^{-1/2} z^2 n_e n_i \nu^{-3} (1 - e^{-\frac{h\nu}{kT}}) \bar{g}_{ff}$

$\kappa^{ff} = \frac{\epsilon^{ff}}{\rho}$

$\frac{\kappa_R^{ff}}{\kappa_{es}} \simeq \frac{0.7 \times 10^{23} \rho T^{-7/2}}{0.4}$ (5.20) (hm 5.2)

有效 opacity $\kappa_* = \sqrt{\kappa_R^{ff} \kappa_{es}}$,　$\tau_* = \rho \kappa_* \Delta s$

同步辐射

极端相对论时 $\omega_B = \frac{qB}{\gamma mc}$, $a_\perp = v_\perp \omega_B$　$\begin{cases} h\nu \gg kT, & 量子为主 \\ h\nu \ll kT, & 都有 \end{cases}$

辐射功率 $P = \frac{2}{3} r_0^2 c \gamma^2 B^2 \beta_\perp^2$,　$r_0 = \frac{e^2}{mc^2}$

各向同性单速电子的平均功率 $\langle P \rangle = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 u_B$,　$\sigma_T = \frac{8}{3}\pi r_0^2$,　$u_B = \frac{1}{8\pi} B^2$

角分布 $\frac{dP}{d\Omega} = \frac{q^2 a_\perp^2}{4\pi c^3} \frac{1}{(1-\beta\mu)^4} \times \left(1 - \frac{\sin^2\theta \cos^2\phi}{\gamma^2(1-\beta\mu)^2}\right)$

存在显著的 beam effect. 功率集中在 $\theta \sim \frac{1}{\gamma}$ 范围

观察到的频谱 $\Delta\omega \simeq \frac{2}{3}\omega_c$

$\omega_c = \frac{3}{2} \gamma^3 \omega_B \sin\alpha$, $\Delta t^A = \Delta t - \frac{\Delta s}{c} \simeq \frac{3}{2}\omega_c^{-1}$

功率谱 $\tilde{P}(\omega) = \frac{\sqrt{3}}{2\pi} \frac{q^3 B \sin\alpha}{mc^2} F\left(\frac{\omega}{\omega_c}\right)$ (depend γ) sometime $\propto \omega^{-s}$,　$P_{tot}(\omega) = \int \tilde{P}(\omega) N(\gamma) d\gamma \propto \omega^{-\frac{(p-1)}{2}}$

电子速度分布 $\propto \gamma^{-p}$

Compton Scattering　频移 $\epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{mc^2}(1-\cos\theta)}$,　$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$,　$\frac{h}{mc} = \lambda_c$

(低能) 散射截面 $= \sigma_T$ + QED 修正

逆Compton散射　$\epsilon : \epsilon' = \frac{1}{\gamma(1-\beta\cos\theta)}$　$\epsilon'_1 : \epsilon'_1 = 1 + \frac{\epsilon'}{mc^2}(1-\cos\Theta)$　$\epsilon_1' : \epsilon_1 = \frac{1}{\gamma(1+\beta\cos\theta_1')}$

$\epsilon : \epsilon' : \epsilon_1' : \epsilon_1 = 1 : \gamma : \gamma : \gamma^2$ ($\gamma\epsilon \ll m_e c^2$)

散射功率 $\frac{dE_1}{dt} = c\sigma_T u_{ph} \left(\gamma^2 (1 + \frac{1}{3}\beta^2) - 1\right) = \frac{4}{3} c\sigma_T u_{ph} \gamma^2 \beta^2$

（光的能量密度）　对电子速度分布积分, thermally $\gamma \simeq 1$

$P_{tot} = n_e \langle P \rangle = \frac{4kT}{mc^2} c\sigma_T n_e u_{ph}$

能量转移: 单次散射 $\Delta\epsilon = \begin{cases} \frac{\epsilon}{mc^2}(4kT - \epsilon) & \text{for Non-rel} \\ 16\epsilon \left(\frac{kT}{mc^2}\right)^2 & \text{for rel} \end{cases}$

平均散射次数 $\bar{N} = \max\{\tau_{es}, \tau_{es}^2\}$,　$\tau_{es} = \rho \kappa_{es} R$ (size)

y parameter $= \bar{N} \times \Delta\epsilon / \epsilon$　　opacity $\simeq \frac{\sigma_T}{m_p} \simeq 0.4\,\text{cm}^2 g^{-1}$

光子的谱, Kompaneets equation:　$\Delta = \frac{\hbar(\omega_1 - \omega)}{kT} \ll 1$, $x = \frac{\hbar\omega}{kT}$,　$= \frac{x \vec{p} \cdot (\vec{n}_1 - \vec{n})}{mc}$

$$\frac{\partial n(\omega)}{\partial t} = c \int d^3p \int \frac{d\sigma}{d\Omega} d\Omega \left[f_e(\vec{p}_1) n(\omega_1)(1 + n(\omega)) - f_e(\vec{p}) n(\omega)(1 + n(\omega_1)) \right]$$

$f_e(E) = n_e (2\pi m kT)^{-3/2} e^{-E/kT}$

$$= c \int d^3p \frac{d\sigma}{d\Omega} d\Omega \left[f_e (1 + \Delta + \tfrac{1}{2}\Delta^2)(n + n'\Delta + \tfrac{1}{2} n'' \Delta^2)(1+n) - f_e n (1 + n + n'\Delta + n'' \tfrac{1}{2}\Delta^2) \right]$$

$$= c\,(n' + n(1+n)) \iint d^3p \frac{d\sigma}{d\Omega} d\Omega f_e \Delta + c\left[\tfrac{1}{2} n'' + n'(1+n) + \tfrac{1}{2} n(1+n)\right] \iint d^3p \ldots$$

$n_e \sigma_T x(4-x) \frac{kT}{mc^2}$　　$2x^2 n_e \sigma_T \frac{kT}{mc^2}$

$t_c \cong n_e \sigma_T c t$

$$\frac{\partial n}{\partial t_c} = \frac{kT}{mc^2} \frac{1}{x^2} \frac{\partial}{\partial x} \left(x^4 (n' + n + n^2)\right)$$

Plasma effect

$$\begin{cases} i\vec{k} \cdot \vec{E} = 4\pi\rho & i\vec{k} \cdot \vec{B} = 0 \\ i\vec{k} \times \vec{E} = \frac{i\omega}{c} \vec{B} & i\vec{k} \times \vec{B} = \frac{4\pi}{c}\vec{j} - \frac{i\omega}{c}\vec{E} \end{cases}$$

$\vec{j} = \sigma\vec{E}$, $\sigma = \frac{i n e^2}{m\omega}$

$\rho = \omega^{-1} \vec{k} \cdot \vec{j}$

$\epsilon = 1 - \frac{4\pi\sigma}{i\omega}$

$$\Rightarrow \begin{cases} i\vec{k} \cdot \epsilon\vec{E} = 0 \\ i\vec{k} \times \vec{B} = \frac{-i\omega}{c} \epsilon\vec{E} \end{cases}$$

色散关系 $k = \frac{\omega}{c}\sqrt{\epsilon} = \frac{\omega}{c}\sqrt{1 - \omega_p^2/\omega^2}$　$\begin{cases} \omega < \omega_p, & \text{forbidden}, \quad x_{decay} \sim 2\pi c/\omega_p \\ \omega > \omega_p, & \text{allowed}, \quad v_{ph} = \frac{\omega}{k} > c, \quad v_g = c^2 k/\omega < c \end{cases}$

Faraday rotation

$\Delta\theta_F = \frac{2\pi e^3}{m^2 c^2 \omega^2} \int_0^d n B_\parallel ds$　$\vec{j} = \frac{i n e^2}{m(\omega \mp \omega_B)} \vec{E}$,　$\omega_B = eB_0/mc$

Bohr: $r_n = \frac{(\hbar c)^2}{e^2 mc^2}$, $a_0 = \frac{\hbar^2}{me^2}$, $v_n = \frac{1}{n}\alpha c < c$, $T_n = \frac{1}{2} m (\alpha c)^2 \frac{1}{n^2}$, $V_n = -2T$

Non-relativistic Quantum Theory of Radiative Process

谱线 { 强度 / 分支系 / 宽度 }　$k = \frac{\omega}{c} \simeq \frac{1}{2}\frac{\alpha}{a_0}$　$ka_0 \ll 1$ 偶极近似

Hamiltonian　$\mathcal{L} = \frac{1}{2} m \dot{x}^2 - q\phi + \frac{q\dot{x}}{c} \cdot \vec{A}$

$\vec{P} = m\dot{x} + \frac{q}{c}\vec{A}$

$\mathcal{H} = \frac{1}{2} m \dot{x}^2 + q\phi$

$$= \frac{1}{2m_e}|\vec{p}|^2 + q\phi - \frac{q}{2mc}(\vec{p} \cdot \vec{A} + \vec{A} \cdot \vec{p}) + \frac{q^2}{2mc^2}\vec{A} \cdot \vec{A}$$

H_{static}　1 photon process　two photon process

$\frac{H_2}{H_1} \ll 1$, $\frac{H_1}{H_0} \ll 1$　(和1 photon 过程按照制的重要)

Helmholtz theorem $\vec{A} = \vec{A}_\parallel + \vec{A}_\perp$

可选gauge使 $\vec{A}_\parallel = 0$　$\{ \nabla^2\phi = -4\pi\rho_e$, static; $\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = 0$, ω

跃迁速率

$H'_{fi}(t) = \langle \phi_f | H' | \phi_i \rangle$　$H'_{fi}(\omega) = \frac{1}{2\pi}\int_0^T H'_{fi}(t) e^{i\omega t'} dt'$　(per unit time) $W_{fi} = \frac{4\pi^2}{\hbar^2 T} |H'_{fi}(\omega_{fi})|^2$

不计偏振 + 偶极近似以下 $\vec{A}(\vec{r}, t) = A(t) \vec{l} e^{i\vec{k}\cdot\vec{r}}$　$e^{i\vec{k}\cdot\vec{r}} \simeq 1$

$$W_{fi} = \frac{4\pi^2 e^2}{T m^2 c^2} |A(\omega_{fi})|^2 \left| \langle f | e^{i\vec{k}\cdot\vec{r}} \vec{l} \cdot \textstyle\sum_j \vec{\nabla}_j | i \rangle \right|^2$$

$$= \frac{4\pi^2 e^2}{m^2 c \omega_{fi}^2} J(\omega_{fi}) \left| \langle f | e^{i\vec{k}\cdot\vec{r}} \vec{l} \cdot \textstyle\sum_j \vec{\nabla}_j | i \rangle \right|^2 = W_{if}$$

$$= \frac{4\pi^2}{c\hbar^2} |(\vec{l} \cdot \vec{d})_{fi}|^2 J(\omega_{fi})$$

$$= \frac{4\pi^2}{c\hbar^2} J(\omega_{fi}) |\vec{d}_{fi}|^2 / 3$$

$B_{lu} = \frac{2}{J(\omega_{ul})} \langle \omega_{ul} \rangle = \frac{8\pi^2}{3c\hbar^2} |d_{ul}|^2$

$A_{ul} = \frac{4\omega_{ul}^3}{3c^3\hbar} |d_{ul}|^2$

有简并时

$B_{ul} = \frac{1}{g_u} \sum_{l,u} B_{ul}^{(ND)}$

$A_{ul} = \frac{1}{g_u} \sum_{l,u} g_{ul}^{(ND)}$

$B_{lu} = \frac{1}{g_l} \sum_{l,u} B_{ul}^{(ND)}$

Absorption oscillator strength emission

$B_{lu} = \frac{4\pi^2 e^2}{\hbar\omega_{ul} mc} f_{lu}$,　$g_l f_{lu} = -g_u f_{ul}$

Selection Rules:　$\Delta S = 0$

Dipole $\begin{cases} \Delta L = 0, \pm 1 & (L = 0 \to 0 \text{ NOT}) \\ \Delta J = 0, \pm 1 & (J = 0 \to 0 \text{ NOT}) \end{cases}$

$d_{fi} \propto \int r \, R_{nl} R_{n'l'} \, dr \int_\Omega Y_{lm}^* Y_{l'm'} \, d\Omega$

Higher　$\Delta J = 0, \pm 1$

Transition Rate　Bound-Bound　$g, f_{in} = \frac{2^9 n^5 (n-1)^{2n-4}}{3(n+1)^{2n+4}}$

Bound-free　$dW = \frac{4\pi^2 e^2}{m^2 c} \frac{c^2}{\omega^2} \frac{dN}{dt dA d\nu} \times |\langle f | e^{i\vec{k}\cdot\vec{r}} \vec{l} \cdot \nabla | i \rangle|^2 \frac{V}{h^3} \times dp\, d\Omega$

$$\frac{dW}{\frac{dN}{dt dA} \cdot d\Omega} = \frac{d\sigma_{bf}}{d\Omega} = \frac{\alpha\nu V}{2\pi\omega} |\langle f | e^{i\vec{k}\cdot\vec{r}} \vec{l} \cdot \nabla | i \rangle|$$

$\sigma_{bf} = \frac{(2\pi)^{9/2} \pi z^5 c^{1/2}}{3 q_0^{3/2} \omega^{7/2}}$, $\hbar\omega \gg R$

$\alpha_0 = n_{atom \text{ at level } n} \sigma_{bf}$

$\psi_0 = \left[\frac{z^3}{\pi a_0^3}\right]^{1/2} e^{-zr/a_0}$