Cosmology

- Standard model of Cosmology = BBT + inflation (→ 1981)

evidence $\begin{cases} \text{Hubble expansion (Hubble 1929 + SN Ia)} \\ \text{CMB (1970s)} \\ \text{BBN} \end{cases}$

- Horizons $\begin{cases} \text{particle horizon} & d_{max}(t) = a(t)\chi(t) = a(t)\int_0^t \frac{c\,dt}{a} \\ \text{event horizon} & d_{Max}(t) = a(t)\int_t^\infty \frac{c\,dt}{a} \end{cases}$

→ 粒子此时 travel 的最远距离

粒子将来能 travel 的最远距离

注意对 $\Omega_\Lambda = 1$ 的宇宙 $a \propto e^t$，不能用 t 积分不能用 a 积分

General Relativity

Space time metric $g_{\mu\nu}(x) \xrightarrow{\text{flat space time}} \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$

线元 $ds^2 = g_{\mu\nu}dx^\mu dx^\nu \begin{cases} >0 & \text{space like} \\ =0 & \text{null} \\ <0 & \text{time like} \end{cases}$ $\quad x'^\mu = R^\mu{}_\nu x^\nu,\ x'_\mu = x_\nu S^\nu{}_\mu$

$R^\mu{}_\nu = \frac{\partial x'^\mu}{\partial x^\nu},\ S^\mu{}_\nu = \frac{\partial x^\mu}{\partial x'^\nu},\ R^\mu{}_\nu S^\nu{}_\sigma = \delta^\mu{}_\sigma$

坐标变换 $T'^{\cdots\mu\cdots}{}_{\cdots\nu\cdots} = R^\mu{}_\rho S^\sigma{}_\nu T^{\cdots\rho\cdots}{}_{\cdots\sigma\cdots}$

Covariant derivative

$$\nabla_\alpha T^{\cdot\cdot\rho\cdot\cdot\sigma\cdot\cdot}{}_{\cdot\cdot\mu\cdot\cdot\nu\cdot\cdot} = \partial_\alpha T^{\cdot\cdot\rho\cdot\cdot\sigma\cdot\cdot}{}_{\cdot\cdot\mu\cdot\cdot\nu\cdot\cdot} - \Gamma_{\alpha\mu}{}^{\lambda} T^{\cdot\cdot\rho\cdot\cdot\sigma}{}_{\cdot\cdot\lambda\cdot\cdot\nu\cdot\cdot} - \Gamma_{\alpha\nu}{}^{\lambda} T^{\cdot\cdot\rho\sigma\cdot\cdot}{}_{\cdot\cdot\mu\cdot\cdot\lambda\cdot\cdot} + \Gamma_\alpha{}^\rho{}_\lambda T^{\cdot\cdot\lambda\cdot\cdot\sigma\cdot\cdot}{}_{\cdot\cdot\mu\cdot\cdot\nu\cdot\cdot} + \Gamma_\alpha{}^\sigma{}_\lambda T^{\cdot\cdot\rho\cdot\cdot\lambda\cdot\cdot}{}_{\cdot\cdot\mu\cdot\cdot\nu\cdot\cdot}$$

$\Gamma^\alpha{}_{\mu\nu} = \Gamma^\alpha{}_{\nu\mu}$ (minimal)

$\nabla_\alpha g_{\mu\nu} = 0$

$\Gamma^\beta{}_{\mu\nu} = \frac{1}{2}g^{\beta\lambda}\{-g_{\mu\nu,\lambda} + g_{\lambda\mu,\nu} + g_{\nu\lambda,\mu}\}$

$\begin{cases} \text{线性性质} \\ \text{Leibniz law} \\ \nabla_\alpha\nabla_\beta \text{ 不对易} \end{cases}$

Curvature

$(\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu)A_\beta = -R^\alpha{}_{\beta\mu\nu}A_\alpha$

$R^\alpha{}_{\beta\mu\nu} = \partial_\mu\Gamma^\alpha{}_{\nu\beta} - \partial_\nu\Gamma^\alpha{}_{\mu\beta} + \Gamma^\alpha{}_{\mu\rho}\Gamma^\rho{}_{\nu\beta} - \Gamma^\alpha{}_{\nu\rho}\Gamma^\rho{}_{\mu\beta}$

前后分别反对称 $R_{\alpha\beta\gamma\delta} = -R_{\alpha\beta\delta\gamma}$

$R_{\alpha\beta\gamma\delta} = -R_{\gamma\delta\alpha\beta}$

$R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta} = 0$

$\nabla_\alpha R_{\mu\nu\beta\gamma} + \nabla_\beta R_{\mu\nu\gamma\alpha} + \nabla_\gamma R_{\mu\nu\alpha\beta} = 0$ (Bianchi)

Einstein Equation

$G_{\mu\nu} = 8\pi G T_{\mu\nu}$ (其中 $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, $R = g^{\mu\nu}R_{\mu\nu}$

$R_{\mu\nu} = \Gamma^\alpha{}_{\mu\nu,\alpha} - \Gamma^\alpha{}_{\mu\alpha,\nu} + \Gamma^\alpha{}_{\beta\alpha}\Gamma^\beta{}_{\mu\nu} - \Gamma^\alpha{}_{\beta\nu}\Gamma^\beta{}_{\mu\alpha}$)

Geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

RW 度规

$ds^2 = -dt^2 + a^2(t)(d\vec{x}^2 \pm dz^2)$

$= -dt^2 + a^2(t)\left(d\vec{x}^2 + K\frac{(\vec{x}\cdot d\vec{x})^2}{1-K\vec{x}^2}\right)$

$= -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2d\Omega^2\right)$

可以将 R 吸收进 a(t)，选择 $R\vec{x} = \vec{x}'$，$K/R^2 = K'$ 是新的空间曲率参数

Cosmological Redshift

$1+z = \frac{\lambda}{\lambda_{rest}} = \frac{1}{a}$, $dz = \frac{-1}{a^2}da$

Distance

Comoving d_c (LOS) $\quad \chi = \int_t^{t_0}\frac{c\,dt}{a} = \int_a^1\frac{c\,da}{a^2H} = \int_0^z\frac{c\,dz}{H}$

Comoving Transverse d_r $\quad r = S_k(\chi) = \begin{cases} \sin\chi, & k=1 \\ \mathrm{sh}\,\chi, & k=-1 \\ \chi, & k=0 \end{cases}$

Angular diameter distance $\quad d_A = ar = a\,S_k(\chi)$

Luminosity distance $\quad d_L = \frac{r}{a} = \frac{1}{a}S_k(\chi)$

Time

$t = \int_0^t dt = \int_0^a\frac{da}{aH} = \int_z^\infty\frac{dz}{H(1+z)}$

$t_{Lookback} = \int_0^z\frac{dz}{(1+z)H}$

Conformal time

$d\eta = \frac{dt}{a} \Rightarrow$ RW metric $ds^2 = a^2(-d\eta^2 + ds_{3D}^2)$

Einstein equation

$\Gamma^0{}_{ij} = a\dot{a}\tilde{g}_{ij}\quad \Gamma^i{}_{0j} = \frac{\dot{a}}{a}\delta_{ij}\quad \Gamma^i{}_{jl} = \tilde{\Gamma}^i{}_{jl} = K\tilde{g}_{jl}x^i$

$R_{ij} = -[2K + 2\dot{a}^2 + a\ddot{a}]\tilde{g}_{ij}\quad R_{00} = 3\frac{\ddot{a}}{a},\quad T^\mu{}_\nu = \begin{pmatrix} -\rho & \\ & p\delta_{ij} \end{pmatrix}$

$\begin{cases} \dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) & \rightsquigarrow \rho = a^{-3(1+w)}\cdot\rho_0 \\ \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G\rho}{3} \end{cases}$

radiation, $w = 1/3$
vacuum E, $w = -1$
matter, $w = 0$

① $\rightarrow \left(\frac{H}{H_0}\right)^2 = -\frac{K}{H_0^2a^2} + \frac{8\pi G}{3H_0^2}\rho$

$= \Omega_K a^{-2} + \Omega_m a^{-3} + \Omega_R a^{-4} + \Omega_\Lambda = E^2$

$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3P)$

其中 $\Omega_K = \frac{-K}{H_0^2}$, $\Omega_m = \frac{\rho_m^0}{3H_0^2/8\pi G} = \frac{\rho_m^0}{\rho_{cr}}, \ldots$

现在观测

$\Omega_K + \Omega_m + \Omega_R + \Omega_\Lambda = 1$

中微子等 重子物质 暗能量

$P = -\rho$ for vacuum energy

ρ remains constant